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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 174

THE SMALL ANGULAR OSCILLATIONS OF AIRPLANES IN STEADY FLIGHT

By F. H. NORTON



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AERONAUTICAL SYMBOLS.

1. FUNDAMENTAL AND DERIVED UNITS.

	Symbol.	Metric.		English.	
		Unit.	Symbol.	Unit.	Symbol.
Length...	l	meter.....	m.	foot (or mile).....	ft. (or mi.).
Time.....	t	second.....	sec.	second (or hour).....	sec. (or hr.).
Force.....	F	weight of one kilogram.....	kg.	weight of one pound.....	lb.
Power...	P	kg. m/sec.....		horsepower.....	HP
Speed.....		m/sec.....	m. p. s.	mi/hr.....	M. P. H.

2. GENERAL SYMBOLS, ETC.

Weight, $W = mg$.

Standard acceleration of gravity,

$$g = 9.806 \text{ m/sec.}^2 = 32.172 \text{ ft/sec.}^2$$

Mass, $m = \frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.³ = 0.07635 lb/ft.³

Moment of inertia, mk^2 (indicate axis of the radius of gyration, k , by proper subscript).

Area, S ; wing area, S_w , etc.

Gap, G

Span, b ; chord length, c .

Aspect ratio = b/c

Distance from $c. g.$ to elevator hinge, f .

Coefficient of viscosity, μ .

3. AERODYNAMICAL SYMBOLS.

True airspeed, V

Dynamic (or impact) pressure, $q = \frac{1}{2} \rho V^2$

Lift, L ; absolute coefficient $C_L = \frac{L}{qS}$

Drag, D ; absolute coefficient $C_D = \frac{D}{qS}$.

Cross-wind force, C ; absolute coefficient

$$C_c = \frac{C}{qS}.$$

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c , D_c .)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

Dihedral angle, γ

Reynolds Number = $\rho \frac{Vl}{\mu}$, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length), C_p .

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w) = \beta$

Angle of attack, α

Angle of downwash, ϵ

REPORT No. 174

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IN STEADY FLIGHT**

By F. H. NORTON
Langley Memorial Aeronautical Laboratory

REPORT No. 174.

THE SMALL ANGULAR OSCILLATIONS OF AIRPLANES IN STEADY FLIGHT.

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SUMMARY.

This investigation was carried out by the National Advisory Committee for Aeronautics at the request of the Army Air Service to provide data concerning the small angular oscillations of several types of airplanes in steady flight under various atmospheric conditions. The data are of use in the design of bomb sights and other aircraft instruments. The method used consisted in flying the airplane steadily in one direction for at least one minute, while recording the angle of the airplane with the sun by means of a kymograph. The results show that the oscillations differ but little for airplanes of various types, but that the condition of the atmosphere is an important factor. The average angular excursion from the mean in smooth air is 0.8° in pitch, 1.4° in roll, and 0.9° in yaw, without special instruments to aid the pilot in holding steady conditions. In bumpy air the values given above are increased about 50 per cent.

INTRODUCTION.

In the design of bombing and navigation instruments it is usually desired to provide a reference platform which will hold as nearly as possible a constant horizontal position. For this purpose it is necessary to know the character of the airplane motions, particularly the small angular ones occurring in steady flight.

Apparently the only data previously available are those incorporated in R. & M. No. 213¹ and R. & M. No. 422.² These tests are quite complete, but were made only in smooth air. They will be referred to again as a means of comparison.

The present tests were made with four types of airplanes at speeds covering the usual flying range and under all air conditions, giving data that should be sufficiently complete to cover the required field.

AIRPLANES AND APPARATUS.

The following airplanes were used in this test:

- (1) A *JN4h* training airplane with standard rigging and normal load. The kymograph was mounted on a rigid support in the rear cockpit.
- (2) A Navy *VE-7* advanced training airplane with standard rigging. The kymograph was mounted on the rear center section strut.
- (3) A standard *DH-4B*.
- (4) A standard Martin bomber without load.

In the last two airplanes the kymograph was mounted on the gun ring.

The approximate characteristics of these four airplanes is given in Table I below for the sake of comparison:

TABLE I.

Airplane.....	JN4h.	VE-7.	DH-4B.	Martin bomber.
Weight during test.....pounds..	2,200	2,100	3,200	10,000
Span.....feet..	43	34	42.5	71
Wing area.....square feet..	350	285	620	1,080
Horsepower.....	180	180	400	800

¹ The Oscillations of an Airplane in Flight and their Effect on the Accuracy of Bomb Dropping.

² Preliminary Tests on the Rolling and Pitching of a Handley Page Machine.

The kymograph used for this test is shown in Figure 1. It was designed to be rigid and compact as it had to be mounted in the air stream. A special type drive³ was developed to give a constant film speed with a very flexible connection to the instrument itself. The record is taken on positive moving-picture film mounted on a revolving drum, the speed of which was in all cases 0.024 inch per second, giving a total time of about 10 minutes for a complete revolution. An angle of 1° is represented on the film by a distance of 0.029 inch. This instrument proved entirely satisfactory throughout the tests.

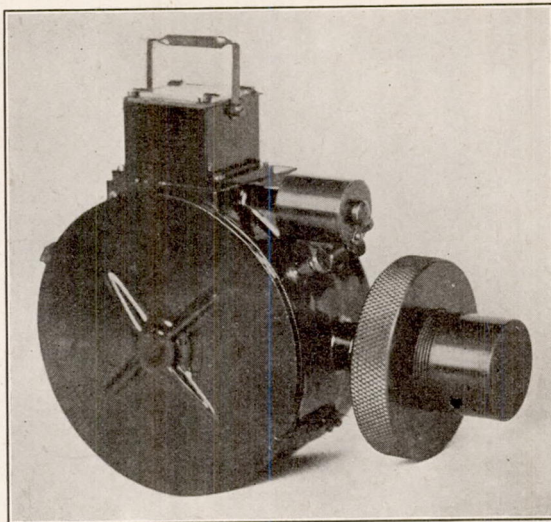


FIG. 1.—N. A. C. A. kymograph

The air speed was measured by the regular air-speed meter in the airplane with no calibration correction. The altitude of the tests varied between 1,000 and 5,000 feet, according to the air conditions. The pilot flew the airplane in each case as he would in approaching a bombing target, and it is probable that the steadiness of flying was not the maximum that could be obtained with extreme care and instrumental guides.

RESULTS.

As the actual kymograph records were very numerous, and as some were too faint for clear reproduction, it was thought confusing to show all; so only a few typical ones are given in Figure 2.

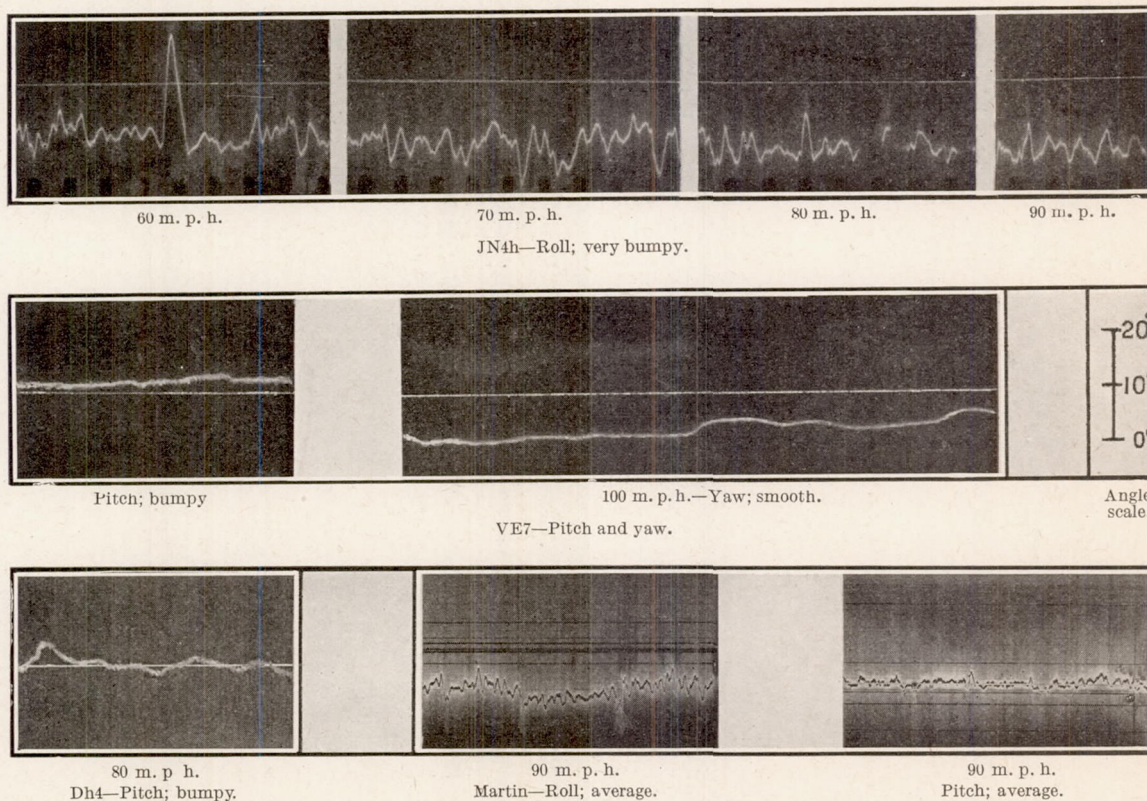


FIG. 2.—Kymograph records.

³ N. A. C. A. Technical Note No. 129, An Impulse Motor for Driving Recording Aeronautical Instruments.

The important characteristics of all of the kymograph curves are summarized in Table II below:

TABLE II.

Motion.	Airplane.	Air condition	Air speed.	Oscillation period.		Displacement from mean.		Angular velocity.		Angular acceleration, maximum.
				Long.	Short.	Maximum.	Average.	Maximum.	Average.	
Pitch.....	JN4h.....	Smooth.....	<i>M. p. h.</i>	<i>Sec.</i>	<i>Sec.</i>	<i>°</i>	<i>°</i>	<i>°/sec.</i>	<i>°/sec.</i>	<i>°/sec.²</i>
Do.....	do.....	do.....	60	12	3	1.0	0.5	0.7	0.1	1
Do.....	do.....	do.....	70	15	6	2.6	1.5	1.2	.2	1
Do.....	do.....	do.....	80	20	3	1.6	1.2	.6	.3	1
Do.....	do.....	do.....	90	22	6	1.5	1.0	.7	.2	1
Do.....	do.....	Bumpy.....	55	12	3	4.0	2.1	1.5	.6	3
Do.....	do.....	do.....	65	14	4	5.0	2.0	2.0	.6	2
Do.....	do.....	do.....	75	14	3	1.2	.9	1.5	.3	2
Do.....	do.....	do.....	85	20	2	3.5	1.5	2.0	.5	3
Do.....	do.....	do.....	95	20	2	2.0	1.1	1.0	.8	3
Do.....	VE-7.....	Smooth.....	70	15	6	1.0	.6	.6	.1	.5
Do.....	do.....	do.....	80	20	5	2.0	1.2	1.2	.2	1
Do.....	do.....	do.....	90	20	4	0.9	.5	.2	.1	1
Do.....	do.....	do.....	100	23	10	1.4	.8	.4	.1	1
Do.....	do.....	Bumpy.....	60	10	3	2.0	.7	1.3	.3	2
Do.....	do.....	do.....	70	15	3	1.7	.7	1.1	.3	3
Do.....	do.....	do.....	80	20	2	2.5	1.0	2.2	.4	3
Do.....	do.....	do.....	90	22	2	2.1	1.2	1.8	.5	2
Do.....	do.....	do.....	100	28	2	2.5	1.3	.7	.3	2
Do.....	DH-4.....	Smooth.....	70	9	4	1.2	.6	.6	.2	.5
Do.....	do.....	do.....	80	14	8	.9	.7	.7	.3	2
Do.....	do.....	do.....	90	17	5	2.2	1.2	.8	.3	1
Do.....	do.....	do.....	100	30	7	1.2	.6	1.1	.2	.5
Do.....	do.....	Average.....	70	11	4	2.0	1.7	.7	.4	2
Do.....	do.....	do.....	80	13	3	2.5	1.2	1.0	.5	3
Do.....	do.....	do.....	90	17	3	2.5	1.5	1.2	.4	3
Do.....	do.....	do.....	100	25	3	1.2	.9	1.5	.3	2
Do.....	do.....	Bumpy.....	70	10	3	1.8	1.1	.9	.4	2
Do.....	do.....	do.....	80	13	4	3.5	1.7	1.4	.4	2
Do.....	do.....	do.....	90	14	3	1.5	.7	.7	.2	3
Do.....	do.....	do.....	100	22	2	2.5	.7	1.1	.3	2
Do.....	Martin bomber.....	Average.....	90	32	2	1.1	1.0	1.0	.2	5
Roll.....	JN4h.....	Smooth.....	60	23	6	2.0	1.0	1.6	.3	2
Do.....	do.....	do.....	70	7	1	6.0	2.5	3.0	1.6	3
Do.....	do.....	do.....	80	5	2	3.5	2.0	3.0	1.2	5
Do.....	do.....	do.....	90	8	1	4.0	2.5	3.5	1.6	5
Do.....	do.....	Bumpy.....	55	8	4	8.0	1.7	7.0	2.1	5
Do.....	do.....	do.....	65	6	4	9.0	2.0	5.0	1.6	7
Do.....	do.....	do.....	75	6	4	5.0	1.7	7.0	1.7	7
Do.....	do.....	Very bumpy.....	60	7	2	4.3	2.1	3.5	2.0	6
Do.....	do.....	do.....	70	7	2	3.5	1.3	2.3	1.1	7
Do.....	do.....	do.....	80	6	1	3.5	1.0	6.0	1.0	8
Do.....	do.....	do.....	90	5	1	2.0	1.0	2.0	1.0	8
Do.....	do.....	do.....	100	18	4	1.3	.9	1.1	.2	1
Do.....	VE-7.....	Smooth.....	60	11	4	2.4	1.0	1.5	.5	2
Do.....	do.....	do.....	70	10	3	1.2	.9	1.1	.2	1
Do.....	do.....	do.....	80	10	3	1.4	.9	1.0	.2	1
Do.....	do.....	do.....	90	14	3	1.4	.9	1.0	.2	1
Do.....	do.....	do.....	100	11	4	1.8	1.3	.9	.2	1
Do.....	do.....	Average.....	80	7	1	3.0	2.0	4.0	1.5	6
Do.....	do.....	do.....	90	10	1	6.0	2.2	2.5	1.3	6
Do.....	do.....	do.....	100	10	1	3.0	1.7	2.5	1.1	6
Do.....	do.....	Bumpy.....	70	7	2	3.4	1.7	2.5	1.0	9
Do.....	do.....	do.....	80	6	1	4.0	2.2	3.4	2.5	10
Do.....	do.....	do.....	90	8	1	4.5	2.0	6.0	1.5	7
Do.....	do.....	do.....	100	7	1	7.0	2.0	4.0	1.1	7
Do.....	DH-4.....	Smooth.....	70	8	2	3.5	1.5	3.7	.7	3
Do.....	do.....	do.....	80	6	2	4.7	2.1	4.0	1.5	5
Do.....	do.....	do.....	90	7	2	2.5	1.4	3.0	.6	5
Do.....	do.....	do.....	100	10	4	1.6	.7	1.2	.3	3
Do.....	do.....	Average.....	70	6	2	6.0	2.0	5.5	1.4	9
Do.....	do.....	do.....	80	6	1	3.0	1.4	3.0	1.1	9
Do.....	do.....	do.....	90	10	1	3.0	1.4	2.5	1.0	7
Do.....	do.....	do.....	100	10	1	6.0	2.1	2.5	1.1	9
Do.....	do.....	Bumpy.....	75	7	2	4.4	1.4	3.5	1.4	7
Do.....	do.....	do.....	85	5	2	6.0	2.0	6.0	2.4	10
Do.....	do.....	do.....	95	5	1	6.0	1.8	4.0	1.0	10
Do.....	do.....	do.....	100	5	2	5.4	1.8	7.0	1.4	8
Do.....	do.....	do.....	105	7	1	3.7	1.4	7.0	1.2	8
Do.....	Martin bomber.....	Average.....	90	6	2	2.8	1.2	3.5	1.0	5
Yaw.....	JN-4h.....	Smooth.....	50	12	3	1.2	1.0	.8	.3	1
Do.....	do.....	do.....	60	14	2	2.0	1.1	.9	.4	1
Do.....	do.....	do.....	75	10	2	2.5	1.1	1.0	.4	1
Do.....	VE-7.....	do.....	70	13	3	3.5	1.8	.6	.3	.5
Do.....	do.....	do.....	80	10	3	5.5	3.2	1.0	.4	.5
Do.....	do.....	do.....	90	8	2	2.8	3.4	1.5	.5	.5
Do.....	do.....	do.....	100	13	2	2.0	1.4	.5	.1	.5
Do.....	DH-4.....	do.....	75	12	4	2.2	.7	.6	.3	1
Do.....	do.....	do.....	85	12	3	2.0	1.1	1.0	.4	1
Do.....	do.....	do.....	95	10	3	1.2	.7	.5	.2	1
Do.....	do.....	do.....	105	10	2	2.2	1.6	1.3	.4	1
Do.....	Martin bomber.....	Average.....	90	10	3	2.0	1.0	.6	.3	3
Pitch.....	Average.....	Smooth.....	(1)	18	6	1.5	.8	.7	.2	1
Do.....	do.....	Average.....	(1)	16	3	2.1	1.3	1.1	.4	2
Do.....	do.....	Bumpy.....	(1)	17	3	2.7	1.2	1.4	.5	2
Roll.....	do.....	Smooth.....	(1)	11	3	2.7	1.4	2.2	.7	2
Do.....	do.....	Average.....	(1)	8	2	4.2	1.8	3.2	1.2	7
Do.....	do.....	Bumpy.....	(1)	6	2	4.8	1.9	4.7	1.5	7
Yaw.....	do.....	Smooth.....	(1)	12	3	2.4	.9	.9	.3	1

¹ Average.

The air condition termed "smooth" signifies air that has only small and infrequent bumps; "average" air is the condition usually prevailing on a bright day at low altitudes and is characterized by small and medium bumps at rather frequent intervals; and "bumpy" air indicates the frequent occurrence of large bumps. It is, however, impossible at the present time to make any quantitative estimate of bumpiness, so that the terms used here give only a rough indication of the air condition.

It was noted that most of the records showed a long and a short period oscillation which appear fairly distinct after careful examination. The period of both the long and the short oscillation is plotted in Figures 3 to 5.

The long period in pitch increases from 10 seconds at 60 miles an hour to about 30 seconds at 100 miles an hour and represents the natural period of the airplane, as can be clearly seen by comparison with the pitching curves given in N. A. C. A. Report No. 170. The short oscillation of from 2 to 4 seconds seems to decrease with an increase in air speed.

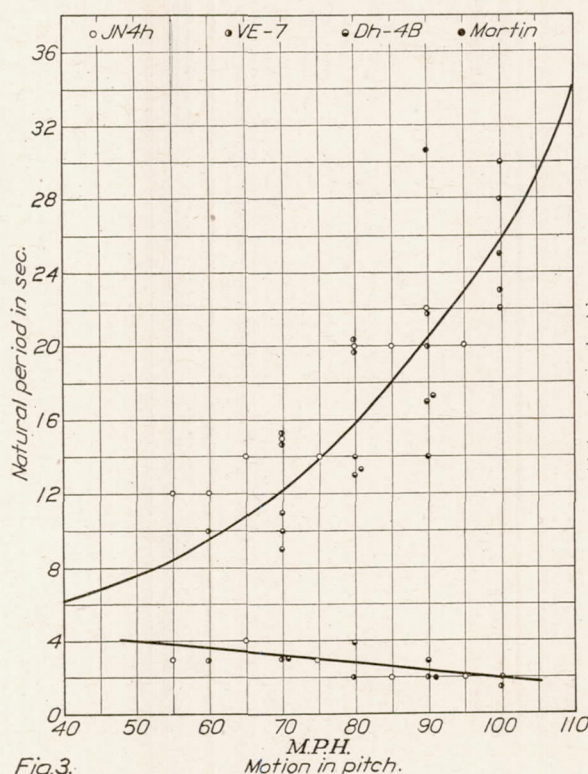


Fig.3.

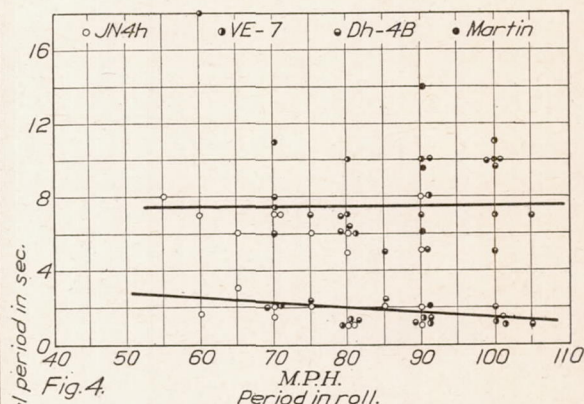


Fig.4.

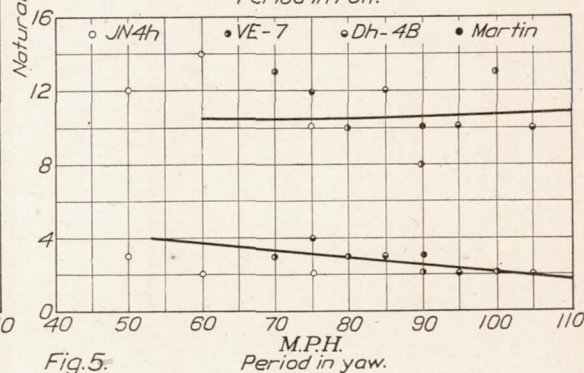


Fig.5.

In yaw and roll the long period is not as definite as in pitch, but it evidently varies only slightly with the air speed and has a period of about 10 seconds. This is probably an oscillation inherent in the airplane. The short-period oscillation is of about the same frequency in pitch, roll, and yaw.

This short oscillation is very interesting, as it indicates a definite air structure with a periodic length of about 300 feet. It should not be concluded that this means a continuous train of vortices of this size, but only that this size distinctly predominates over other sizes. Another explanation may be that the airplane itself has a natural period of two to four seconds and that this oscillation is excited only when it encounters air gusts of approximately that period. In fact, mathematical studies have shown that an airplane has a short period of somewhat this length, but very highly damped, and it is possible that in bumpy air this short period might come into evidence. The series of kymograph records shown in Figure 2 brings out clearly this small oscillation with variation in air speed. If this is caused by air structure and this air structure was found to be uniform from time to time, such records as these might serve to roughly measure the air speed of an airplane or the wind past a stationary object. It is interesting to

note that Mr. Lucas in R. & M. No. 213 states that in bumpy air a short period of $2\frac{1}{2}$ seconds was observed, thus agreeing well with the present values.

The values for long-period oscillation as given in Table II are seen to be in fair agreement with the results published in R. & M. No. 213 and R. & M. No. 422, which are:

Airplane.....	B. E. 2C.	Handley Page.
Pitch.....	Sec. 20	Sec. 25
Roll.....	30	40
Yaw.....	20

Turning now to the amplitude of the oscillation, it was thought best to give the maximum and average angular excursion, referring only to the long oscillation as the others are of small range. The amplitude does not appear to vary appreciably with the type of airplane. However, the largest machine, the Martin bomber, is slightly steadier than the other machines, although no hard and fast conclusions can be drawn from the limited number of runs available on this machine. On the other hand, the amplitude does vary markedly with the air conditions, as can be seen from the averages given at the bottom of Table II. Bumpy air increases the amplitude in all cases about 50 per cent over the conditions in smooth air. On the other hand, the amplitude in roll is about 50 per cent greater than in pitch and yaw, and it seems improbable that it can be held much below 2° of average excursion, and at times it may reach 5° if the air is bumpy.

When the displacements of the small oscillations are examined, it appears that they are of less magnitude at high speed, due to the greater aerodynamic damping, and because each air current has less time to act on the airplane.

Again referring to the British tests, it is seen that the agreement is good:

Airplane.....	B. E. 2C.	Handley Page.
Pitch.....	• 0.6	• 1.0
Roll.....	1.0	2.0
Yaw.....	.6

The angular velocity of the airplane was determined from the slope of the kymograph curve with the appropriate scale corrections. For example, the film speed was 0.024 inch per second and $1^\circ = 0.029$ inch, which for a slope of 1 in 1.05 would give:

$$\frac{1.05}{1} \times \frac{0.029}{0.024} = 1.2^\circ \text{ per second.}$$

The average angular velocity is about the same for all of the airplanes, but it is about twice as large in roll as in pitch and yaw and twice as large in bumpy as in smooth air.

The maximum angular acceleration was found approximately from the minimum radius of curvature at each peak. This radius was measured under the microscope, but due to the width of the line may not be correct to better than 25 per cent. The angular acceleration in degrees per second² can readily be found from the expression:

$$\alpha = \frac{a^2}{b r}$$

where a is the time scale.

b is the angular scale.

and r is the radius of curvature.

The angular acceleration in roll is much larger than in pitch or yaw, and it may reach $10^\circ/\text{sec.}^2$ in bumpy air. This is about 0.2 radian/sec.² and is equal to a sudden movement of the ailerons through 2° angle at 80 miles per hour.

CONCLUSIONS.

The following facts are brought out by an examination of the kymograph records:

(1) The average amplitude in degrees from the mean is:

Air condition.....	Smooth.	Bumpy.
	°	°
Pitch.....	0.8	1.2
Roll.....	1.4	1.9
Yaw.....	0.9	...

(2) The average periodic motion may be considered as divided into one of about 10 seconds' period due to the airplane and to one of about 3 seconds' period due to the air structure or combination of the air structure and airplane. In smooth air the long period predominates, and the short one is indeterminate. In bumpy air the long period is masked and the short period is prominent and varies in frequency directly as the air speed.

(3) The average angular velocity in degrees per second is:

Air condition.....	Smooth.	Bumpy.
	°/sec.	°/sec.
Pitch.....	0.2	0.2
Roll.....	.7	1.5
Yaw.....	.3	...

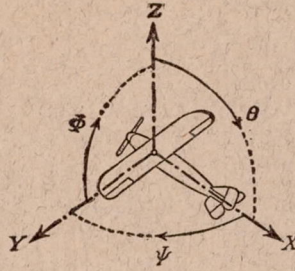
(4) The average angular acceleration in degrees per second, per second is:

Air condition.....	Smooth.	Bumpy.
	°/sec. ²	°/sec. ²
Pitch.....	2	3
Roll.....	8	10
Yaw.....	2	..

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▽



Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Sym- bol.		Designa- tion.	Sym- bol.	Positive direc- tion.	Designa- tion.	Sym- bol.	Linear (compo- nent along axis).	Angular.
Longitudinal....	X	X	rolling.....	L	Y → Z	roll.....	Φ	u	p
Lateral.....	Y	Y	pitching...	M	Z → X	pitch.....	Θ	v	q
Normal.....	Z	Z	yawing.....	N	X → Y	yaw.....	Ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{q b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q f S}$$

Angle of set of control surface (relative to neutral position), δ . (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D

Pitch (a) Aerodynamic pitch, p_a

(b) Effective pitch, p_e

(c) Mean geometric pitch, p_g

(d) Virtual pitch, p_v

(e) Standard pitch, p_s

Pitch ratio, p/D

Inflow velocity, V'

Slipstream velocity, V_s

Thrust, T

Torque, Q

Power, P

(If "coefficients" are introduced all units used must be consistent.)

Efficiency $\eta = T V / P$

Revolutions per sec., n ; per min., N

Effective helix angle $\Phi = \tan^{-1} \left(\frac{V}{2\pi r n} \right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.

1 kg. m/sec. = 0.01315 HP

1 mi/hr. = 0.44704 m/sec.

1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.

1 kg. = 2.20462 lb.

1 mi. = 1609.35 m. = 5280 ft.

1 m. = 3.28083 ft.